The Social Cost of Public Finance

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This paper examines the implications for fiscal policy of systematic differences between (social) rates of return on private investment and savings. We show that the social discount rate lies between these divergent rates of return, that the controversy between this result and Marglin's stems entirely from different treatments of depreciation, and that reinvestment of *net* project output has negligible quantitative effects on the social discount rate. Then, within a macroeconomic framework, we derive the stringent conditions for a unique discount rate and demonstrate that, as public-sector consumption must also be charged a shadow price, the social value of a global change in output induced by countercyclical fiscal policy must exceed its (nonzero) social cost. Finally, we examine the Little-Mirrlees concept of the social cost of labor and find it unacceptable.

I. Introduction

For decades the most controversial issue in cost-benefit analysis has been the selection of the appropriate rate of discount. The controversy initially focused on the relative merits of two candidate rates: (1) the (relatively low) social rate of time preference, which we shall call the consumption rate of interest (r) and which many writers have identified with the after-tax rate of return on private savings; and (2) the (relatively high)

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gross-of-tax rate of return to privately financed investment, which we shall call the investment rate of interest (ρ). Recently, however, a number of analysts, the most significant being Sandmo and Dreze (1971), Harberger (1973*b*), and Dreze (1974), have demonstrated that the social rate of discount (ω) should be a weighted average of ρ and r. As a result, the debate has been slightly refocused, the candidate rates now being r and the weighted average ω.

This paper is an attempt both to resolve the existing controversy and to indicate the importance of a number of additional problematic issues that have not received previous attention in the literature. In Section II*A* we rely on a simplified model of a project which generates a perpetuity to demonstrate that, when capital-market distortions are correctly taken into account, ω equal to a weighted average of ρ and r is the only result that can obtain, even though we also agree that r is the only defensible rate by which alternative future consumption paths should be evaluated. We further demonstrate, again for the case of a perpetuity, that Marglin's (1963*a*, 1963*b*) well-known analysis yields an investment criterion identical to the net-present-value criterion when ω is used as the discount rate.

Then, in Section II*B*, we show that a true difference between Marglin's analysis and those which result in ω does arise for projects with finite lives; the divergence stems from diametrically opposed, implicit assumptions as to consumption behavior toward depreciation of, and net income from, project-created capital. This point leads us, in Section II*C*, directly into the question of reinvestment; arguing that Marglin's results, although theoretically correct, depend on an ad hoc treatment of depreciation while the Harberger[1] and Sandmo-Dreze[2] models do not directly address the issue,[3] we present two alternative manners by which reinvestment of the project's *net* addition to output can be incorporated into the analysis.

At another level of analysis, however, it is our contention that the debate on the social rate of discount has to date neglected a basic issue underlying the evaluation of public *expenditures*. In particular, concentration on investment criteria and the rate of discount has led, at least implicitly, to an asymmetric treatment of government consumption versus government investment. This has in turn, we argue, obscured the one fundamental question that must be resolved: What is the social opportunity cost of *any* type of government expenditure?

Sections III and IV explore this question within the context of a simple

[1] Our repeated references to the Harberger approach to the social discount rate will consistently be in terms of the 1973*b* article.

[2] We will generally refer to the Sandmo-Dreze model as the body of analysis contained in both the Sandmo-Dreze (1971) and the Dreze (1974) articles.

[3] Harberger implicitly assumes that it is handled elsewhere in the evaluation of the project itself; Sandmo-Dreze avoid it in their 1971 paper by their "sudden-death" assumption.

macroeconomic model. We first note the obvious point that when changes in public expenditures alter voluntary, privately financed expenditure decisions—investment or consumption—there is a clearly associated, and in principle directly observable, opportunity cost which may or may not require reference to information from the capital market. We then show how all of the relevant considerations can be combined to formulate a procedure for socially evaluating (any multiplier effects of) *general* public expenditures, the more convenient manner being a Marglin-type shadow price of inputs.

Finally, in Section V, we evaluate the social opportunity cost of labor as expounded by Little and Mirrlees in the OECD Manual (1969) and which has remained largely intact in their more recent publication (1974). Our analysis has direct relevance to this seemingly unrelated issue since their shadow price of labor depends crucially on a distinction between the investment and consumption rates of interest. We conclude, as have others, that the Little-Mirrlees analysis is wrong, and for a reason even more fundamental than reasons which earlier critics have cited.

II. The Social Rate of Discount, the Opportunity Cost of Public Investment, and Two Distinct Reinvestment Issues

A. Derivations of ω When Public Projects Generate Perpetuities

Controversy over the appropriate rate of discount would never arise in an undistorted economy, as there would exist only one relevant (market) rate of interest, simultaneously representing the (social) marginal efficiency of investment and the consumption rate of interest. Using this rate to fulfill the net present value criterion for any publicly financed project would therefore guarantee that the project is capable of compensating private investors and consumers for both the privately financed current consumption and the potential consumption that they are induced to forgo by the initiation of the government expenditure.

The issue, of course, arises because capital markets are almost universally distorted; a variety of taxes (and other distortions) on the income from capital and on the yields from savings drives ρ upward and r downward, respectively, in relation to "the" market rate of interest. In this context, those who argue for the exclusive use of ρ—which in fact represents the social (i.e., tax-inclusive) return to private investment—rely on a pure concept of opportunity cost; discounting public expenditures by any rate of interest less than ρ raises the distinct possibility that a publicly accepted project could yield smaller *social* returns than would directly investing in the private sector. On the other hand, those who argue for the exclusive use of the consumption rate of interest rely on the following reasoning. The objective of fiscal policy should be to maximize social welfare, approximated by an aggregate utility function which depends only on the time stream of consumption. Since r is defined by the private valuation of current relative to future consumption—that is, r is the supply price of savings—it can be the only relevant rate for discounting the benefits generated by public expenditure.

Both of these positions are correct, in and of themselves, yet each ignores the validity of the other. There can be no doubt that r is the correct rate for discounting positive and negative increments to future consumption. But likewise, there can be no doubt that current public expenditure must be charged not only with current consumption forgone but also with unrealized potential future consumption due to displacement of current investment in other sectors. The obvious implication is that we should seek a social discount rate that encompasses both of these factors; the result will necessarily be a rate ω that lies between ρ and r.

To show this result, we assume a closed economy in which all shadow prices, other than the social rate of discount, equal their market prices, and consider a public-sector investment of ΔI^g that generates a perpetuity. The resultant (permanent) change in output, when private investment also generates perpetuities, is defined as: $\Delta Y = \rho \Delta I^p + \delta \Delta I^g$, where Y is output, I^p is private investment, and δ is the realized rate of return on the public project. The operator Δ refers to the departure of any variable from the path it would have followed had the public project not been undertaken, rather than to changes that occur from one period to another.

In any economic system, with or without an organized capital market, each dollar of ΔI^g takes place at the expense of some fraction (possibly zero) θ of investment and $(1 - \theta)$ of consumption. Without loss of generality, we set ΔI^g equal to unity; hence $\Delta I^p = -\theta\rho$, and $\Delta Y = \delta - \theta\rho$. Clearly the criterion for accepting the project must be that total income available for consumption be at least as great with the project as without it. Since we are explicitly utilizing marginal analysis, it is appropriate to compare the present value of ΔY, discounted at r, with consumption forgone, $(1 - \theta)$. That is, the investment criterion is:

$$\int_0^\infty (\delta - \theta\rho)e^{-rt}\,dt \geq (1 - \theta).$$

Solving for δ, we obtain:

$$\delta \geq \theta\rho + (1 - \theta)r \equiv \omega. \qquad (1)$$

Obviously the social rate of discount, ω, is a weighted average of ρ and r, the weights being the aforementioned shares of increments to public expenditure that come at the expense of investment and consumption, θ and $(1 - \theta)$.

The one remaining issue is the definition of these weights, for which we fall back on Harberger's pioneering analysis, and on its subsequent confirmation and extension by Sandmo and Dreze. That is, departing from an initial equilibrium position in a distorted capital market, Harberger demonstrates that the (perpetual) stream of income required to compensate private-sector investors and consumers for a (permanent) extraction of \$1.00 from that market is:[4]

$$\omega = \left[\rho\left(\frac{\partial I}{\partial i}\right) + r\left(\frac{\partial C}{\partial i}\right)\right] \Big/ \left[\left(\frac{\partial I}{\partial i}\right) + \left(\frac{\partial C}{\partial i}\right)\right].$$

This weighted average formulation highlights Harberger's major contribution toward resolving the discount-rate controversy; given a perfect but distorted capital market and rational economic behavior, it is the interest rate which allocates output between consumption and investment. Therefore, the capital market interactions that occur in response to government-borrowing-induced changes in the interest rate define the shares of each dollar of public investment that come at the expense of privately financed investment

$$\left[\frac{\partial I}{\partial i} \Big/ \left(\frac{\partial I}{\partial i} + \frac{\partial C}{\partial i}\right)\right]$$

and consumption

$$\left[\frac{\partial C}{\partial i} \Big/ \left(\frac{\partial I}{\partial i} + \frac{\partial C}{\partial i}\right)\right].$$

These capital-market-determined weights, then, are our θ and $(1 - \theta)$, respectively.

Harberger's analysis has been criticized for two alleged inadequacies, first, that it shares the same problems of all analyses based on concepts of consumers' and producers' surplus. This point is obviously invalid; even a cursory reading indicates that Harberger's graphical analysis is merely a pedagogical guide to the intuition behind his derivation of ω. A second criticism is that the model is not of a sufficiently "general equilibrium" nature. There are three senses in which this latter point has some substance. First, Harberger assumes private-sector maximization behavior, rather than explicitly incorporating it into his analysis. Second, and more important, the approach is concerned only with "sourcing"—the raising of funds—although the manner in which they are to be spent can clearly

[4] Of course, Harberger's results extend to any number of distorted investment and/or consumption activities that might be affected by the change in government borrowing. For our purposes, however, such an elaboration is unnecessary.

affect private expenditure decisions.[5] And third, no account is taken of income/saving relationships.

Recent and independent work by Sandmo and Dreze (SD) has produced results identical to Harberger's, eliminating, in their 1971 paper, the first substantive objection above. Using a two-period analysis of utility-maximizing consumers and profit-maximizing firms, SD show that, if the government's objective is to choose its level of investment so as to maximize the economy's utility function, subject to the government's second-period budget constraint, the resulting first-order condition is:

$$g'(z) = \frac{(1 + r)[(\partial C/\partial r)|_u] + [1 + r/(1 - t)](\partial y/\partial r)}{[(\partial C/\partial r)|_u] + \partial y/\partial r} = (1 + \rho),$$

where (in their notation) $g(z)$ is the public-sector investment function, r is the consumption rate of interest, C is first-period consumption, t is the rate of tax on profits, y is the level of privately financed investment, and ρ is defined as the social rate of discount. Thus, relying on the same capital-market mechanisms, SD also find that ω is the *same* weighted average (in our notation, $\rho = r/(1 - t)$ and $1 - \theta = [(\partial C/\partial r)|_u]/\{[(\partial C/\partial r)|_u] + \partial y/\partial r\}$).

Although the SD framework shares the latter two substantive shortcomings of Harberger's model, at the same time their work clarifies several particular, common features of these three formulations of ω. First, the proper weight for the consumption rate of interest is explicitly shown to be the income-compensated, government-borrowing-induced impact on current consumption. Sandmo and Dreze explain this implication by noting that the overall budget constraint forces interest payments deriving from government borrowing to be mere transfers. More generally, it is by now a standard implication of economic theory (see Bailey 1957)[6] that if a (marginal) change in the interest rate is caused solely by fiscal or monetary policy, with no accompanying change in real resources or investment opportunities, the community cannot experience a change

[5] Notice that we are not directly referring here to the uniqueness convention adopted by Harberger because of the "familiar [problem] in public finance, where one can develop a separate analysis of any tax for each possible way the proceeds can be spent, and a separate analysis of each possible expenditure for each possible way the money can be raised" (Harberger 1973*b*, p. 111). Rather, the issue involved (see Sec. III below) is the broader macroeconomic one concerning the general public's perceptions of and therefore their (voluntary) expenditure-reactions to (*a*) the value of public output and (*b*) the degree to which bond and tax finance are viewed as equivalent. Hence, analyses which have attempted to derive the social rate of discount by tracing down private expenditures displaced via incremental taxation are subject to this same limitation (see Krutilla and Eckstein 1958; Subcommittee on Economy in Government of the Joint Economic Committee 1968; Haveman 1969).

[6] As the substitution effect in question arises because the economy is constrained to movements along the transformation function, that effect is slightly different from the Slutsky substitution effect which envisages movements along an indifference surface.

in real income, and any observed change in consumption occurs only through the substitution effect.

Second, although it has been alleged that the weighted-average concept of ω holds only for a two-period world, Dreze has shown in his "postscriptum" (1974) that the same result obtains in the multiperiod case. Dreze also argues that the source of finance for public investments does not alter the formula for ω.

To reiterate, then, if both ρ and r are correctly incorporated into the calculation of the social discount rate, the only possible result is an ω that lies between the (inclusive) limits of ρ and r. However, the existence of Harberger's demonstration (to take the first) of this fact has not removed the controversy, as proponents of the exclusive use of r have not yet been convinced. Perhaps the analyst most often cited in support of this latter position is Stephen A. Marglin, particularly with respect to his two papers, "The Social Rate of Discount and the Optimal Rate of Investment" (1963*a*) and "The Opportunity Costs of Public Investment" (1963*b*).

One substantive difference—the definition of r—between models that generate ω and Marglin's approach can be dismissed as immaterial in the current context. For Marglin (1963*a*), r is the "social rate of time preference" whereas Harberger and SD presume that r is best approximated by the net-of-tax yield on savings. Whether the social rate of time preference systematically differs from (an appropriately weighted average of) after-tax rates of returns to saving will not be examined in what follows as the precise choice of r can affect only quantitative results. As we are concerned with the conceptual issue of defining the *social* discount rate in the context of capital market distortions, it is sufficient for our purposes simply to view r as the appropriate rate for discounting future consumption. For simplicity, however, we shall continue to treat r as the after-tax yield on savings.

Aside from this relatively minor point—minor in terms of the controversy as to the appropriate social rate of discount—reliance on Marglin's system as an alternative to analyses which yield ω is misplaced. For perpetuities the two approaches yield identical investment criteria. Marglin's general expression for the net present value of a public-sector project is:

$$\int_0^\infty B(x, t)e^{-rt}\,dt - \alpha K(x),$$

where B measures future (net) project benefits as a function of the scale of investment (x) and time, K is the initial period capital cost, and α is the social opportunity cost per dollar (shadow price) of public investment. If the economy is undistorted, $\alpha = 1$; if there are capital market distortions, however, $\alpha > 1$ is an "adjustment factor" to reflect the fact that each dollar of public investment will displace some fraction θ—a fraction to which Marglin does not give an operational definition—of private investment, and that this displaced investment carries a potential stream of consumption whose present value exceeds $\$\theta$.

In order to determine α, Marglin defines ρ as the (perpetual) gross (or social) yield on private investment; then the present value of this perpetuity forgone is simply $\theta\rho/r$. Assuming output constant, the fraction $(1 - \theta)$ of every dollar of public investment comes from current private consumption, and since everything is measured in terms of that consumption, no adjustment is required for the term $(1 - \theta)$.[7] Therefore, the shadow price of a dollar of public investment is simply:

$$\alpha = \theta\rho/r + (1 - \theta). \tag{2}$$

Now when the public project is a perpetuity, $B(x, t) = B(x)$, and Marglin's net present value criterion becomes: $B(x)/r \geq \alpha K(x)$. But since K is totally expended during the initial period, this criterion carries the implicit discount factor $r\alpha$—that is, $B(x)/r\alpha \geq K(x)$—and, obviously, $r\alpha = \theta\rho + (1 - \theta)r = \omega$.

Therefore, Marglin's full system is equivalent to analyses demonstrating that ω is the appropriate rate of discount.[8] Analysts aligned with Marglin have concentrated (and we are convinced that, on this point at least, Marglin would be in full agreement) solely on r as the rate for determining the present value of potential future consumption—B/r and $\theta\rho/r$—without paying adequate attention to all of the implications of his correct formulation of α. Under our interpretation, r is in fact the discount rate with which consumers evaluate alternative time paths of consumption. However, in the presence of capital-market distortions, initiation of a publicly financed project (generating a perpetuity) will have three separate consequences to be taken into account: (1) displaced private-sector consumption; (2) displaced private investment, and therefore $\theta\rho$ of (potential) future consumption in perpetuity; and (3) the creation of a new perpetual income stream of δ. The correct procedure is to discount both streams 2 and 3 by r. But, for the marginal project ($\delta = \omega$) our procedure of discounting by ω guarantees that, in fact, we discount only the net change in potential future consumption by r. Hence, the correct investment criterion is obtained either by Marglin's shadow pricing of capital or by discounting the net benefit stream by ω.

[7] Marglin, Harberger, and Sandmo-Dreze all use current consumption as their *numeraire*; Little and Mirrlees, however, inadvertently use investment as *numeraire*, and do so erroneously.

[8] The *ordering* of present values will be the same under the Marglin and Harberger, Sandmo-Dreze approaches. In the case of Marglin, $PV_M = B(x)/r - \alpha K(x)$, whereas the Harberger, Sandmo-Dreze present value is $PV_{H-SD} = B(x)/\omega - K(x)$. As $\alpha r = \omega$, $PV_{H-SD} = (r/\omega)PV_M$.

B. Public-Sector Projects Have Finite Lives: The Treatment of Depreciation

Marglin's analysis and those that generate ω as the appropriate discount rate are equivalent only when public projects yield perpetuities. For projects with finite lives, the two approaches diverge significantly; the reason is subtle and fundamental, but has absolutely nothing to do with the appropriate rate of discount. Rather, the divergence stems from the assumed treatment of gross-of-depreciation output. Marglin's model contains neither an operative capital market nor net private-sector investment; hence, all output produced by the capital stock must be consumed. Now when public-sector projects generate perpetuities, there is no depreciation and the consumption behavior implicit in this model does not imply net consumption of capital (be it public or private).

However, consider the opposite "polar" case—a project all of whose benefits $(1 + \delta)$ appear exactly one period after the investment is made. Marglin's net-present-value criterion (with $K = 1$) still holds:[9]

$$\int_0^\infty B(x, t)e^{-rt}\,dt \cong B(x, 1)/(1 + r) = (1 + \delta)/(1 + r) \geq \alpha.$$

Here, the implicit discount factor is $\alpha(1 + r) = \omega + \alpha$; that is, one dollar of public investment now yields $(1 + \delta)$ of gross benefits next year, the present value of which must exceed $(1 - \theta) + \theta\rho/r$, implying that $\delta \geq \omega + (\alpha - 1)$.

On the other hand, the social-discount-rate approach results in the same criterion, $\delta \geq \omega$, as in the case of a perpetuity. This result is consistent with the Harberger-SD assertion that ω is the unique social rate of discount, regardless of the source of project financing. They base their assertion on the assumption of general access to the capital market, which yields up and absorbs the changes in financial resources caused by public (or private) expenditure activities. But in order to obtain this result, an additional, more specific assumption concerning consumption behavior—one which is only implicit in the Harberger-SD analyses—must be made. Of the gross project output, $1 + \delta$, one dollar is clearly depreciation. Hence, the project's contribution to net income in the second period is δ less $\theta\rho$—the income forgone on displaced private investment. The necessary assumption is that consumers do not treat public-project depreciation as income subject to current consumption, but instead intend to save (and hence reinvest) all of that depreciation.

Now this intended saving (i.e., a rightward shift of the savings function) will be partially frustrated, injections of funds into the capital market having effects exactly symmetric with extractions. That is, introduction of the \$1.00 of depreciation into the capital market for intended saving

[9] The equality between $B(x, 1)/(1 + r)$ and the integral is only approximate, as r in the former refers to discrete time whereas r in the latter is defined on continuous time.

will result in only $\$\theta$ of actual investment, since the accompanying fall in the market rate of interest will lead would-be savers to consume $\$(1 - \theta)$ instead. Therefore, the intended reinvestment of all public-project depreciation will actually increase (private) investment by only θ in period one. But this is precisely the amount of investment required to return the total capital stock, and the future income from that stock, to the paths that would have existed if the project had never been undertaken. Hence, given this fundamental assumption, the investment criterion (for a marginal project) simply involves comparing the (potential) increase in consumption in period one with the consumption actually forgone in period zero. No other relevant changes take place. The criterion is, therefore, $[(1 - \theta) + (\delta - \theta\rho)]/(1 + r) \geq (1 - \theta)$, or that $\delta \geq \theta\rho + (1 - \theta)r \equiv \omega$.

Thus, in the social-discount-rate approach, the useful life of the project is irrelevant, given the conditions postulated, whereas in Marglin's shadow-price approach the length of the project's life is crucial. In fact, contrary to outward appearances that must derive from a narrow concentration on the rate of discount, Marglin's criterion—$\delta \geq \omega + (\alpha - 1)$—is more stringent than analyses which discount by ω (recall that $\alpha > 1$). The reason for this superficial anomaly is that, in his model, people treat the gross return from the project—that is, the net return δ plus the depreciation on the publicly created capital (said depreciation being 100 percent in the polar case)—as income which they intend to, and in fact do, consume. The result is a global capital stock θ dollars lower after completion of the project than it would have been had the project never been undertaken. Hence, to be acceptable, the project must not only cover the social rate of discount but must also pay a penalty sufficient to compensate for the social opportunity cost of the capital that is consumed *as a direct consequence of the project*. Plausible assumptions concerning parameters indicate that the penalty could easily be as high as 75 percent of the invested capital.[10]

Although we have compared Marglin's approach with that yielding ω as the social rate of discount for just the two polar cases of perpetuities and two-period projects, the comparative results generalize to any investment profile.[11] The crucial difference between the approaches is not the rate of discount but rather the implicit assumption about the intended allocation of depreciation between consumption and investment. If Marglin's assumption holds, the above implications clearly follow. On the other hand, it is our contention that individuals maximizing welfare over time will attempt to distinguish net income from depreciation, and hence will not intentionally consume depreciation simply because it is available. It is this explicit behavioral assumption that distinguishes (and extends) our understanding of a well-functioning capital market from that of Harberger and Sandmo-Dreze.

At least two qualifications are in order at this point. First, general access to the capital market as a means of disposal of depreciation is an important restriction on the finding that ω is the social rate of discount.[12] Second, while it is perhaps relatively simple for owners of private firms to distinguish depreciation from net output, it is by no means obvious that consumers can do so with the same ease when the output is produced in the public sector.[13] If consumers have no way of identifying depreciation and if the government does not intervene with reinvestment, then consumers will likely, if unwittingly, consume depreciation in its entirety, and we find ourselves in the world of Marglin's simplest model. At the other extreme is the idea implicitly underlying the social-discount-rate approach, that consumers do find it possible to identify correctly the depreciation component of gross output. Persons who argue that the truth lies somewhere in the middle must be prepared to accept the direct implication that the rate of discount must be higher the shorter is the life of the project, as short-term projects will generally lead to capital consumption at an earlier date than will long-lived projects.

Whatever position is accepted, our analysis indicates that reinvestment does critically influence the outcome when projects have finite lives; the reason is not that saving is superior or inferior to consumption (at the margin), but rather that saving is transformed by the capital market into private-sector investment which earns a rate of return greater than the consumption rate of interest.[14] The reinvestment issue is clearly

[10] E.g., suppose that $\theta = 0.75$, $\rho = 0.10$, and $r = 0.05$, implying that $\alpha = 1.75$ and $\omega = 0.0825$. Then, under Marglin's explicit criterion and its underlying behavioral assumptions, one dollar of public investment must not only yield 8.25 percent to cover the social discount rate, but also yield an additional \$0.75 to compensate the economy for the capital consumption (as opposed to creation!) caused by the project. In effect, Marglin's model generates an implicit tax of 75 percent on public investment.

[11] Since the time profile of costs and benefits for any project can be derived from the time profiles of a series of two-period projects, the above results are easily generalized. A proof is provided in an appendix, available from the authors upon written request.

[12] Although we are purposely abstracting from distribution issues in this paper, we do recognize that some or all of the recipients of public-project benefits may not avail themselves to the capital market. If some beneficiaries are consuming all of their income and would consume more if possible, presumably at least some of the project's depreciation will be consumed in its entirety without being channeled back to the capital market.

[13] The ambiguity concerning depreciation is precisely the problem that comes up in the context of project profiles with multiple internal rates of return. This issue is treated in the appendix referred to in n. 11.

[14] Even in the case of a perpetuity there is no a priori reason to believe that all of the increase in net output is consumed. Even though our criterion requires that the present value of future consumption be at least as large as $(1 - \theta)$ in order to have no worsening of welfare, once the project is in fact undertaken the community faces a new opportunity set. We cannot assume that people will choose to consume all of the project's net output, but if they do so it is clear that they will have been exactly compensated for the initial consumption forgone. If they voluntarily consume less than the increment to net income—

twofold; first, how the community intends to distribute—and ultimately does distribute—depreciation between consumption and (re)investment; and second, the treatment of increments to net income created by a public-sector project. We now turn to this second issue.

C. Reinvestment of Net Project Output

While our discussion of the first reinvestment issue—one that has apparently not received previous attention in the literature—has revealed that the difference between the Marglin and the Harberger-SD approaches is at once superficial (for perpetuities) and fundamental (for finite-lived projects), our examination of the second issue—the allocation of "new" income between consumption and saving—exposes a conceptual weakness in the social-discount-rate approach. Although both Harberger and SD posit positive private-sector saving and investment, neither explicitly explores the implications of the fact that an acceptable public-sector investment must produce a change in income—and hence potential investment—in at least one period. On the other hand, Marglin devotes a great deal of his formal analysis to this issue. We find it pointless, however, to examine his models in detail because he treats privately financed investment (and saving) as being totally independent of capital-market phenomena; that is, in his model(s) private investment passively conforms to saving, which in turn is determined in an ad hoc manner without reference to the capital market.[15]

It has been argued that a well-functioning capital market renders current public-investment decisions logically independent of future investment possibilities. However, if project-induced reinvestment has measurable capital-market effects—if it alters the path of privately financed investment—then the social rate of discount can be required to take these effects into account. We shall illustrate an appropriate procedure for doing so, again with the two polar cases of a perpetuity and a two-period project. For both cases, we assume that the public sector invests \$1.00 in period zero and nothing thereafter.

For the case of a perpetuity, the public project generates the following effects in the investment period: $\Delta C_0 = -(1 - \theta)$, $\Delta I_0^p = -\theta$, $\Delta I_0^g = 1$, where the subscripts refer to the time period.[16] Continuing, $\Delta Y_1 = \delta - \theta\rho$, $\Delta I_1^g = 0$, $\Delta I_1^p = \Delta I_1 = \theta(m\Delta Y_1)$, $\Delta C_1 = \Delta Y_1(1 - \theta m)$, m being the marginal propensity to save and $m > 0$ indicating that positive reinvestment is occurring. Then: $\Delta Y_2 = \Delta Y_1 + \rho\Delta I_1 = \Delta Y_1(1 + m\theta\rho)$, $\Delta I_2 = m\theta\Delta Y_1(1 + m\theta\rho)$, and $\Delta C_2 = (1 - m\theta)\Delta Y_1(1 + m\theta\rho)$. During period three: $\Delta Y_3 = \Delta Y_1 + \rho(\Delta I_1 + \Delta I_2) = \Delta Y_1(1 + m\theta\rho)^2$, $\Delta C_3 = (1 - m\theta)\Delta Y_1(1 + m\theta\rho)^2$. For the nth period, we have: $\Delta Y_n = \Delta Y_1(1 + m\theta\rho)^{n-1}$, and $\Delta C_n = (1 - m\theta)\Delta Y_1(1 + m\theta\rho)^{n-1}$. The present value, as of period zero, of this stream of changes in consumption is:

$$-(1 - \theta) + [(\delta - \theta\rho)(1 - m\theta)/(1 + r)][1 + (1 + m\theta\rho)/(1 + r) + (1 + m\theta\rho)^2/(1 + r)^2 + \cdots],$$

and assuming that $r > m\theta\rho$, the series converges to: $-(1 - \theta) + (1 - m\theta)(\delta - \theta\rho)/(r - m\theta\rho)$. For the above expression to be nonnegative, it is required that:

$$\delta \geq (\omega - m\theta\rho)/(1 - m\theta) \equiv \omega_1. \qquad (3)$$

Equation (3) defines ω_1 as the "reinvestment adjusted" social rate of discount. When $m = 0$, $\omega_1 = \omega$, and given that $\rho > \omega$ it follows that ω_1 declines relative to ω as m increases: $\partial\omega_1/\partial m = -\theta(\rho - \omega)/(1 - m\theta)^2$. As we shall see, this adjustment is likely to be very small.

Turning to the two-period project, we assume that benefits in period one are $(1 + \delta)$ and nothing thereafter. Assuming that depreciation equal to \$1.00 during period one is seen as part of gross but not net income and that society therefore attempts to save it all by placing it in the capital market, the project effects during period zero and one are:

$$\Delta C_0 = -(1 - \theta), \quad \Delta I_0^p = -\theta, \quad \Delta I_0^g = 1;$$

$$\Delta Y_1 = (\delta - \theta\rho), \quad \Delta I_1^g = 0, \quad \Delta I_1^p = \Delta I_1 = \theta + m\theta\Delta Y_1;$$

$$\Delta C_1 = (1 + \Delta Y_1) - \Delta I_1 = (1 - \theta) + (1 - m\theta)\Delta Y_1.$$

Thereafter the changes in the flows proceed as rather simple series:

$$\Delta Y_2 = -\theta\rho + \rho\Delta I_1 = m\theta\rho\Delta Y_1,$$

$$\Delta I_2 = m\theta\Delta Y_2 = \rho(m\theta)^2\Delta Y_1,$$

$$\Delta C_2 = (1 - m\theta)\Delta Y_2 = (1 - m\theta)(m\theta\rho)\Delta Y_1;$$

$$\Delta Y_3 = -\theta\rho + \rho(\Delta I_1 + \Delta I_2) = m\theta\rho(1 + m\theta\rho)\Delta Y_1,$$

$$\Delta C_3 = (1 - m\theta)\Delta Y_3 = (1 - m\theta)(m\theta\rho)(1 + m\theta\rho)\Delta Y_1,$$

the general terms for ΔY and ΔC being:

$$\Delta Y_n = (m\theta\rho)(1 + m\theta\rho)^{n-2}\Delta Y_1,$$

$$\Delta C_n = (1 - m\theta)(m\theta\rho)(1 + m\theta\rho)^{n-2}\Delta Y_1.$$

and such a choice is obviously voluntary rather than dictated by a budget (or by some other, artificial) constraint—then the presumption must be that they consider themselves to be better off as a consequence. Hence, our procedure of comparing the present value of change in future income with current consumption forgone implies a sufficient condition for project acceptance.

[15] Indeed Marglin at one point identifies a reinvestment parameter with the marginal propensity to save (1963*b*, p. 282).

[16] Recall that the operator Δ refers to differences measured from the values that would have existed in the absence of the publicly financed project.

The present value of this stream of changes in consumption is given by the sum: $-(1-\theta) + [(1-\theta) + (1-m\theta)\Delta Y_1]/(1+r) + (1-m\theta)(m\theta\rho)\Delta Y_1/(1+r)^2 + (1-m\theta)(m\theta\rho)(1+m\theta\rho)\Delta Y_1/(1+r)^3 + \cdots$, which, again assuming that $r > m\theta\rho$, converges to: $-[r/(1+r)] \cdot [(1-\theta) - (1-m\theta)(\delta - \theta\rho)/(r - m\theta\rho)]$. Requiring this expression to be nonnegative results in precisely the conditions that lead to equation (3): $\delta \geq (\omega - m\theta\rho)/(1-m\theta) \equiv \omega_1$.

This definition of ω_1 as the reinvestment-adjusted social rate of discount naturally adapts to Marglin's concept of a shadow price of inputs, which corresponds perfectly with his definition of that shadow price under identical assumptions:

$$\alpha_1 \equiv \omega_1/r = [\omega/r - m\theta\rho/r]/(1-m\theta)$$
$$= [(1-\theta) + (1-m)\theta\rho/r]/(1-m\theta).$$

As this result may be generalized to multiperiod projects (refer to n. 11), the adjusted social rate of discount is invariant with respect to the life or profile of the publicly financed project.

In the above derivation of ω_1 and α_1, we have taken a slightly different measure of benefits than in Sections II*A* and II*B*, where we discounted changes in output rather than consumption; this conceptual difference affects the social rate of discount when the marginal propensity to save is nonzero. Since either approach can be defended, it is not clear which is superior, although discounting consumption is closer to the spirit of second-best analysis. But that analysis is itself at once paternalistic and schizophrenic; one may well ask what sense it makes for the government, in its expenditure decision, to take into account the fact that investment is more valuable socially than is consumption when that inequality has in fact been introduced by the government in its taxation policy? To put it another way, if people are blissfully unaware of externalities, do we really do them a favor by taking such effects into account in the public sector, and if they are in fact aware of externalities but choose to do nothing about them, are these really and truly externalities? Because consumption is the ultimate objective of investment, one feels "right" about treating the (discounted value of) variations in consumption as benefits; on the other hand, if people are unaware of the excess of the gross rate of return on investment (ρ) over the consumption rate of interest (r), they will view saving and consumption as equally valuable at the margin and hence their welfare is in some relevant sense "best" measured by the path of output.

Because reinvestment introduces a larger absolute difference in the output than in the consumption path, discounting reinvestment-induced changes in the former leads to a larger downward adjustment in the

social rate of discount. Letting ω_1' be the social discount rate adjusted for the effect of reinvestment on output, we derive:[17]

$$\omega_1' = \omega - (1-\theta)m\theta\rho. \qquad (3')$$

Having demonstrated that reinvestment does have a qualitative effect on the social rate of discount, whether we measure the adjustment relative to the change in consumption or output, we hasten to add that the magnitude of the adjustment is likely to be quite small.[18] It is also observed that, for given θ and m, the positive impact of ρ on ω is larger than the adjustment (in absolute value) on either ω_1 or ω_1'. While logical consistency compels that one or the other of these reinvestment adjustments be taken into account, the fact of the matter is that its quantitative significance is marginal. This "practical" result derives from the fact that the reinvestment effect under consideration arises solely from the increment to net income (consumption); in contrast with Marglin, we have already assumed that people attempt to save all depreciation from public-sector investments.

It might be objected, however, that regardless of the size of the adjustment, deducting it from the social rate of discount is inappropriate since this would tend to bias the analysis in favor of longer-lived and more capital-intensive projects.[19] In fact there is an alternative method for dealing with reinvestment; since they are directly induced by the project, reinvestment effects can obviously be treated as "external" additional benefits to be attributed to the project, with the augmented benefit stream then being discounted by ω.[20]

[17] From the definitions of ω_1 and ω_1', it is straightforward to derive the difference $(\omega_1 - \omega_1') = m\theta(1-\theta)(r - m\theta\rho)/(1-m\theta)$, which is positive because the discounting procedure requires that $r > m\theta\rho$. Thus $(\omega - \omega_1) < (\omega - \omega_1')$.

[18] Suppose that $\theta = 0.8$, $m = 0.1$, $\rho = 0.2$, and $r = 0.05$; then the output-reinvestment adjustment to ω is -0.32 percent and the consumption-reinvestment adjustment is -0.26 percent. If we take the implausibly low value of 0.5 for θ and the rather high value of 0.3 for m, the output-reinvestment adjustment is -1.32 percent. Finally, assuming a more realistic value of 0.12 for ρ (and 0.3 and 0.5 for m and θ, respectively) the output-reinvestment adjustment is -0.9 percent and that for consumption is -0.6 percent.

[19] This same objection—in reverse, of course—is frequently encountered in arguments against including even an appropriate risk premium in the social rate of discount; the objection is as invalid in this case as in the case of our reinvestment adjustment. While it is, of course, true that lower (higher) rates tend to make longer-lived projects more (less) acceptable, there is no *artificial* bias involved either with the reinvestment adjustment to the social rate of discount or with the addition of an appropriate (project-by-project) risk premium. For an excellent treatment of risk premia and the rate of discount, see Bailey and Jensen (1972).

[20] It is this treatment of benefits that is implied by Harberger's (1973*b*) "sourcing" argument. For a more direct treatment of the reinvestment issue in terms of "external" effects, a treatment which in certain respects anticipates at least the spirit of our approach, see Harberger (1973*c*).

Nevertheless, it is to be emphasized that this latter procedure is a strictly equivalent alternative to discounting the nonaugmented benefit stream by ω_1. Discounting by ω_1 is appropriate if the benefits from reinvestment are not included in the calculation of a given project's net benefit stream; if these additional benefits are included, discounting by ω is appropriate. The important point is that our extra term in the social rate of discount—or in the net benefit stream—arises solely because the project causes consumption to fall in the initial period and income and consumption to rise in at least one following period; reinvestment arises, and cannot be ignored, precisely because (and only because) not all of the increase in income need be immediately consumed.

The last point to be made in this section, one that is rather obvious but nevertheless suffers neglect, concerns the permanency of the effects of public-sector investment. One of the key difficulties of fiscal policy in general is that the public nearly always has the capability to offset or negate any effect of government action by altering its own consumption and/or investment behavior. In the case of public-sector investment, a marginal project ($\delta = \omega$) will alter the voluntarily chosen paths of private consumption and investment only if the project is a perpetuity or if it generates reinvestment. Otherwise, consumption will fall by $(1 - \theta)$ and private investment by θ in the initial period: subsequently, consumption will increase by precisely the amount necessary—$(1 + r)(1 - \theta)$ in the two-period case—to compensate consumers, and private investment will be replenished by θ, leaving the path of future income unaltered. That the effects of such a project are purely transitory is guaranteed by our concepts of gross and net income, plus the existence of a well-functioning capital market.

III. The Social Opportunity Cost of Public Funds

One of the more important results from the previous section is the general equivalence of shadow pricing capital-good inputs à la Marglin and discounting future flows at the social opportunity cost of capital. We now turn to an examination of these concepts within the broader framework of public finance and expenditure in general, in contrast to the more narrow focus of both prior approaches as far as they have been developed in the literature. The process reveals the additional costs and benefits to be included in a comprehensive cost-benefit analysis of all public expenditure, be it for consumption or investment purposes.[21]

While neither the social-discount-rate approach nor the shadow pricing of inputs has an a priori claim as the theoretically superior concept for encompassing macroeconomic effects, the latter turns out to be the more amenable approach. Of course, the rationale for shadow pricing is that the financing of public investment may directly affect privately financed investment and consumption activities.[22] But if public consumption "projects" also affect the capital market, does it not follow that one must similarly shadow price inputs devoted to public consumption? Put another way, under what circumstances is the apparent asymmetrical treatment of public investment and consumption projects found in so much of the public finance literature logically substantial?

In a direct and obvious sense, funds devoted to public-sector consumption and investment projects have identical costs; funds spent on consumption could have been used for investment and, if acceptable investment projects are lacking, there is always the possibility of retiring government debt (or buying into the capital market in general) which will have a social yield of ω and hence a shadow value of α. This shadow value is the opportunity cost of using existing funds for consumption purposes and, as we have seen, there is reason to believe that α is well above unity. Thus, on these grounds at least, it seems that the case for penalizing investment projects either by charging a high shadow price for capital-good inputs or by requiring a rate of return greater than the consumption rate of interest is simultaneously a justification for insisting that the benefits of publicly financed consumption exceed the costs thereof and do so by a wide margin. There seems to be no obvious reason on the grounds of opportunity cost, then, for the asymmetric treatment of investment and consumption that is so deeply entrenched in the traditions of public finance.

If a case does exist for asymmetric treatment of publicly financed consumption and investment, it must lie in the possibility that these two types of expenditure have differential effects on privately financed spending. If, for example, the public sector began distributing consumer goods already supplied and demanded in the private sector, then we may anticipate that privately financed consumption will decline (private-sector saving rise) by a magnitude similar to the increase in either taxes or bond sales required to finance the new expenditure if other outlays are to be undisturbed. The social cost of this type of expenditure would be quite different from one in which privately financed expenditure had to be contracted by a rise in the rate of interest.

To examine the consequences of public expenditure of the sort referred

[21] For simplicity we now abstract from the reinvestment issue in order to concentrate our attention on other aspects.

[22] There are two types of shadow pricing in this context; the first is always applicable to project inputs and outputs and is necessary because of distortions in the markets in which *these goods* are traded. The second type of shadow pricing—the one we associate with Marglin—is in our framework a further adjustment on prices of inputs arising from distortions in the market for *financial* capital. Only this second adjustment is of concern in this paper.

to in the foregoing paragraph, it is convenient to introduce a simple macroeconomic model. This approach has an additional advantage in that it permits us to investigate certain assumptions underlying both the shadow price and the social rate of discount approaches treated in Section II.

The model employed distinguishes between private and public expenditure and also between the cost and value of the latter. The following notation is used: y is real output, y^* is perceived real income of the private sector, f is real private-sector acquisition of goods and services, including the perceived value of public-sector output, g is the cost of public-sector output, t is real tax revenue minus transfers (including interest payments) of the public sector, and b is the rate of net real bond sales by the public sector. Perceived real income of the private sector is defined by:

$$y^* = y + h(g) - t - \phi(b), \qquad h' \geq 0, \qquad \phi' \geq 0, \tag{4}$$

where h is the private-sector valuation of public-sector output, whether that output be consumption or capital goods. Ideally, the argument of the h function should be a vector, as h' may assume a distinct value for each of the many different kinds of goods provided by the public sector. For simplicity of exposition we will retain the mathematical form of equation (4) but we will treat the value of h' as a variable. The function ϕ represents the value people place on future tax liabilities required to service the public debt. Obviously h involves the perceptions of the public and ϕ their intentions with respect to bequests to future generations.

The budget constraint of the public sector is:

$$g = t + b, \tag{5}$$

and that of the private sector:

$$f = y - t - b + h(g). \tag{6}$$

Acquisitions by the private sector, $f(y^*, r, g)$, include the value of goods and services received in kind from the public sector; these goods and services must be consumed or held—and hence acquired—in order to be realized as income. Thus, privately financed expenditure is $f - h(g)$, and the budget constraint for the entire economy is:

$$f + g = y + h(g). \tag{7}$$

Finally, total expenditure is defined as:

$$y^e = f - h + g. \tag{8}$$

The monetary sector of the economy is not explicitly introduced at this point, but we do assume that the demand for real cash balances depends upon real output (y) and the consumption rate of interest (r).[23]

[23] As inflation is not contemplated in the model, it matters little that r is a real rate of interest.

Finally, the investment and consumption rates of interest are assumed (for simplicity) to be linked by a positive constant such that $d\rho = dr$.

In the context of cost-benefit analysis, it is generally assumed that public expenditure comes at the expense of private consumption and investment *at a given rate of real output*. Thus, when considering conventional cost-benefit analysis in this macroeconomic context, we assume that the authorities adjust the money supply to maintain contemporary aggregate demand (y^e) at a constant level, even in the face of changes in public finance and expenditure.

Now, given the above model, the effect on the interest rate of simply extracting funds from the capital market can be found by differentiating equation (8) partially with respect to b and setting the result equal to minus unity:[24] $\partial y^e/\partial b = -1 = f_{y^*}(-\phi') + f_r(\partial r/\partial b)$; $f_{y^*} > 0$, $f_r < 0$.

Thus:

$$\partial r/\partial b = (\phi' f_{y^*} - 1)/f_r, \tag{9}$$

where f_{y^*} is the marginal propensity of "spend" and $f_r = \partial C/\partial r + \partial I/\partial r$.[25] The interest rate effect of tax finance is obtained in the same manner: $\partial y^e/\partial t = -1 = -f_{y^*} + f_r(\partial r/\partial t)$, or:

$$\partial r/\partial t = (f_{y^*} - 1)/f_r. \tag{10}$$

Equality of equations (9) and (10) is tautological when $\phi' = 1$; that is, if there is no "burden" of the debt. The proposition that tax and bond finance of government expenditure may have identical effects is one that has long been a part of public-finance literature and tradition.[26]

If increases in the public debt are to pose no "burden" for future generations, it must be true that the citizenry fails to distinguish between tax and bond finance in determining its consumption and investment behavior. If this be the case, it obviously follows that tax multipliers are exactly zero.[27]

Returning to equation (9), it is central to all three derivations of the social discount rate, ω (Section II), that the effect of sourcing on the rate of interest be $(-1/f_r)$; we see that this will be true only if f_{y^*} or ϕ' is zero.

[24] In order for the funds to be obtained at a constant real output, saving must rise relative to investment by exactly one unit, and hence aggregate expenditure must decline by one unit.

[25] We assume for simplicity that public-sector spending is completely interest inelastic so that we can continue to interpret C and I as privately financed consumption and investment, respectively. Note also that $\theta = (\partial I/\partial r)/f_r$.

[26] This proposition has recently been brought once again to the attention of monetary economists by Barro (1974). While Barro's technical treatment of the issue is no less than novel, it leads to no new insights (see, e.g., Bailey 1962, or Hause 1964).

[27] The equivalence of tax and debt finance is also argued by Harberger and Sandmo-Dreze in connection with the social opportunity cost of capital. Their position arises from the explicit (or implicit) assumption that investment in the capital market is a permissible use of tax revenues.

Assuming $f_{y^e} \neq 0$, there appears to be no compelling logic nor empirical evidence supporting the extreme assumption that ϕ' is zero. On the other hand, if $\phi' = f_{y^e} = 1$, then $\partial r/\partial b = 0$ because all new bond issues are voluntarily purchased at the expense of private consumption and/or investment without changes in the interest rate. This extreme sensitivity of spending behavior with respect to the size of the public debt places debt and tax finance on equal footing insofar as interest-rate effects are concerned and, if this were indeed the case, the social rate of discount would change drastically.[28] Assuming that the direct effect of bond finance falls entirely on consumption, we have:[29] $\partial C/\partial b = -\phi' f_{y^e} + (\partial C/\partial r)(\partial r/\partial b)$, and from this result it is straightforward to derive the social discount rate as: $r[1 - \theta(1 - \phi' f_{y^e})] + \rho\theta(1 - \phi' f_{y^e})$. If $\phi' = 1$, the weight received by ρ is $\theta(1 - f_{y^e})$ rather than θ; the effects are dramatic as presumably f_{y^e} is a rather large fraction, perhaps near unity.[30]

At this point it is convenient to examine the Harberger (1973*b*, p. 111) and Dreze (1974, p. 60) contention that tax and bond finance have equal social opportunity costs. Their argument does not require identical interest-rate effects associated with the two sourcing operations but rather is based on the idea that the option of debt retirement or direct investment in the capital market constitutes a viable alternative use of tax revenue, and that this option would yield a social benefit identical with the social cost of extracting funds from the capital market. That is, while the total social cost of raising funds may well depend upon the mode of finance, the opportunity cost of the *use* of those funds is determined solely in the capital market.[31] For this argument to be correct, it is necessary that the interest-rate effect of taxation be zero—that is, that the cost of using tax funds for a project consist only of the loss of benefits that could be obtained by investing those funds in the capital market.[32] In terms of equation (10), the requirement is that f_{y^e} be exactly unity; if this is not the case—if taxation itself has interest rate effects—then the option of placing tax

[28] *Insensitivity* of privately financed expenditure to the size of the debt implies either a failure to perceive the debt itself or an unsatisfied desire for negative bequests. We assume that negative bequests reduce welfare, since the transfer away from future generations is clearly involuntary on their part. For a terse but excellent treatment of the negative bequest issue, see Hause (1964).

[29] This is derived by partially differentiating (8) with respect to b and positing that *all* changes in expenditure appear as changes in (privately financed) consumption.

[30] The marginal propensity to spend can, of course, exceed unity. One may argue that its magnitude is affected by the length of run, being smaller in the short run. This is not necessarily true if the public fully anticipates the benefits and hence the income associated with project execution and the future costs of bond finance.

[31] This argument violates the idea that the sourcing of funds is independent of their subsequent use; indeed, recognition that the social opportunity cost of capital depends upon alternative use rather than just sourcing destroys the presumptive uniqueness of the social discount rate. The logic of the "alternative use" argument clearly requires that the effects of both the expenditure and the receipt of funds be taken into account. This point is treated further later in this section.

[32] We are neglecting both direct collection and resource-misallocation costs in the case of tax finance.

funds in the capital market cannot generally yield an accurate measure of the cost of tax finance.

Within the Harberger and Sandmo-Dreze sourcing framework, then, sufficient conditions for uniqueness of the social opportunity cost of public-sector funds, ω, are (*a*) that the marginal propensity to spend be unity and (*b*) that the public be totally insensitive to the size of the public debt ($\phi' = 0$). Although one cannot rule out the possibility that these conditions are in fact met in the real world, it remains true that there exists neither a logical nor an empirical case for the choice of these particular values for f_{y^e} and ϕ'.

Further complications arise when we consider the effect of project execution; if expenditure, as well as sourcing, has effects on the capital market, these effects must somewhere be taken into account. One way of doing so is to define, for each project, a distinct social cost of capital that includes the capital-market effects of both the finance and execution of the project. A second way is to define a unique social cost of capital and adjust the cost and benefit streams to reflect all ramifications of project execution, including capital-market effects. A third, and perhaps the most straightforward, way is to discount by the consumption rate of interest but to shadow price inputs in order to incorporate additional costs and benefits emanating from the capital market as a consequence of both project financing and execution. The second method is in the spirit of the Harberger and Sandmo-Dreze approach, while the third corresponds to Marglin's. As all three methods, in principle, give identical results, any preference for one over the others is clearly a matter of taste.

We now proceed to determine the implications for the social rate of discount of directly incorporating the capital-market effects of project execution. Turning first to the interest-rate effect, we equate y and y^e, differentiate equation (8) totally, set the result equal to zero, and obtain:

$$dr/dg = -(1/f_r)\{f_{y^e}[(1 - \phi')\, db/dg - (1 - h')] + 1 + f_g - h'\}. \tag{11}$$

The interpretation of all terms of (11) is obvious, apart from f_g. That term represents the direct effect of public outlays on privately financed expenditure; that is, it reflects the broadening of the opportunity set for both private consumption and investment as a consequence of increased public-sector expenditure. It does not reflect any competitive displacement of privately financed consumption or investment by public-sector activity; that effect is fully captured in the final term $(-h')$ of equation (11).[33] Hence, f_g is assumed to be nonnegative.

[33] This is easily seen by assuming that the public sector produces a good already privately purchased; if tax financed, we expect that total expenditure will remain constant. Holding output (y), the interest rate (r), and government borrowing (b) constant, the change in total expenditure is: $dy^e = f_{y^e}(-dg + h'dg) + f_g dg - h'dg + dg$. If the government is perceived to produce its output as efficiently as the private sector, then $h' = 1$ and $dy^e = f_g dg = 0$, if $f_g = 0$.

Under bond finance, $db/dg = 1$, so equation (11) becomes:

$$dr/dg = -(1/f_r)[1 + f_{y^*}(h' - \phi') - h' + f_g]; \quad (12)$$

the tax finance case is easily shown to be identical to (12), with unity replacing ϕ'. With bond finance, sufficient conditions for dr/dg to equal $(-1/f_r)$ are: (*a*) contemporaneous real output constant; (*b*) no direct effect on private expenditure: $f_g - h' = 0$; and (*c*) equal perceptions: $h' = \phi'$. Conditions (*b*) and (*c*) together imply that $h' = \phi' = f_g$, which seems plausible only if each term is zero. This is the spirit of the Harberger, Sandmo-Dreze sourcing approach, in which the supply schedule of funds available for privately financed investment is shifted by the full amount of the public-sector bond issue.[34] Once again, the conditions are sufficiently demanding that they are highly unlikely to be fulfilled in practice, and hence we expect that project financing and implementation will have capital-market effects that in turn imply costs and/or benefits above and beyond those captured by ω.

Let us now return to the question of the shadow price for consumption versus investment projects in the context of bond finance. The social value of the output of a consumption project is clearly h', from which we must deduct the present value of any additional future taxes required to replace revenue lost due to displaced private-sector investment. On the other hand, the present value of additional future taxes required to service the increment in the debt is clearly unity; hence perceived income is reduced by ϕ' on this account.[35] In the case of an investment project, the debt service remains ϕ', and again we must deduct from the present value of the perceived future benefits, h', any additional taxes required to compensate for revenue forgone due to displaced, private-sector investment. This symmetry between consumption and investment projects does not, however, permit us to establish any relationship between ϕ' and h', as the former depends upon perceptions of and attitudes toward future taxes, while the latter is influenced by the characteristics of the project itself and the parameters of the system.

The shadow price of inputs will be obtained by determining the minimal acceptable level of h'. We begin by noting that there are three distinct sources of costs associated with financing and executing a project. The first arises (mainly) because of the direct displacement of private consumption and investment, the second comes about because of the consumption and investment effects of broadening the opportunity set, and the third has its origin in the interest-rate effects. Letting $y^{e*} = f - h$

[34] Sufficient conditions for dr/dg to equal $-(1/f_r)$ under tax finance are both (*a*) and (*b*), and that $h' = 1$; i.e., all project output is "saved" and used to pay the increment in current taxes.

[35] Taxes causing the market rate of interest (i) to differ from r do not affect the argument as the *net* interest paid on public debt is obviously r.

be privately financed expenditure, total displacement of y^{e*} under bond finance is given by: $dy^{e*}/dg = -1 = [f_{y^*}(h' - \phi') - h'] + f_g + f_r dr/dg$. The first term, $f_{y^*}(h' - \phi') - h'$, which measures the change in privately financed consumption and investment because of changes in perceived income as well as substitution of public for privately financed output, is assigned a shadow price of $\alpha^* = \theta^*\rho/r + (1 - \theta^*)$, θ^* being the fraction of $f_{y^*}(h' - \phi') - h'$ accounted for by changes in private investment. The second term, f_g, has a shadow price of $\alpha' = \theta'\rho/r + (1 - \theta')$ where θ' is that fraction of f_g appearing as new investment; finally, the term $f_r dr/dg$ has the usual shadow price of α. Introducing a negative sign into each term to represent cost rather than benefit, we define the social cost, α_2, as: $\alpha_2 = \alpha^*[-f_{y^*}(h' - \phi') + h'] + \alpha'(-f_g) + \alpha(-f_r dr/dg)$. Substituting equation (12) for the final term and rearranging, we have:

$$\alpha_2 = \alpha + (\alpha - \alpha^*)[f_{y^*}(h' - \phi') - h'] + (\alpha - \alpha')f_g. \quad (13)$$

The total cost defined in equation (13) is thus the social opportunity cost of public funds (neglecting the reinvestment adjustment developed in Section II*C*), but is itself a function of h', the present value of the project output. To eliminate h' from the expression, we simply require that $h' \geq \alpha_2$, and solve (13) as follows:

$$h' \geq \frac{\alpha - (\alpha - \alpha^*)f_{y^*}\phi' + (\alpha - \alpha')f_g}{1 + (\alpha - \alpha^*)(1 - f_{y^*})}. \quad (14)$$

Alternatively, if $\delta = rh'$ is the rate of return, we require

$$\delta > \frac{\omega - (\rho - r)[(\theta - \theta^*)\phi' f_{y^*} - (\theta - \theta')f_g]}{1 + (\rho/r - 1)(\theta - \theta^*)(1 - f_{y^*})} = \omega_2, \quad (15)$$

where ω_2 is the social opportunity cost of capital in this context.

Inequality (14) expresses the investment criterion in terms of the minimum value of output per dollar of input—by definition the shadow price of inputs. Inequality (15) expresses the same criterion in terms of the rate of return, δ.

As a simple case of expression (14), let us consider a consumption project, for which it is reasonable to assume that $\theta^* = \theta$; any direct substitution effects are likely to fall only on privately financed consumption. Further, assume that this project—an ABM site?—broadens neither the consumption nor the investment set: $f_g = 0$. Finally, if f_{y^*} is unity, the shadow price becomes:[36]

$$h' \geq \phi' + \alpha(1 - \phi'), \quad (14')$$

[36] Fixing f_{y^*} equal to unity only implies that, in the context of tax finance, there will be no hoarding of public-sector output. That output causes privately financed expenditure to fall directly by h' but to rise by $f_{y^*}h'$; the tax increase directly reduces expenditures by an additional f_{y^*}. The net change is $f_{y^*}(h' - 1) - h'$; if $f_{y^*} = 1$, the net change is minus unity, which implies no hoarding.

or a weighted average of α and unity. Similarly simplified, expression (15) becomes:

$$\delta \geq r\phi' + \omega(1 - \phi'), \tag{15'}$$

a weighted average of r and ω.

The critical coefficient is ϕ', which lies in the zero-unit interval. If we assume that $\phi' = 0$, then our conditions become merely $h' \geq \alpha$, and $\delta \geq \omega$, which are the Marglin and Harberger, Sandmo-Dreze definitions of the shadow cost of capital and the social rate of discount, respectively. The striking aspect of this result is that it refers to a consumption project: the conditions that lead us to accept ω as the social rate of discount are precisely those that *require* us to use α as the shadow price of bond-financed, public-sector consumption.[37] This conclusion is a direct but apparently infrequently appreciated implication of both the shadow-pricing and the social-discount-rate approaches.

If, on the other hand, $\phi' = 1$, expressions (14) and (15) become: $h' \geq 1$, and $\delta \geq r$. In this case, there is no need to shadow price inputs and the social rate of discount becomes merely the consumption rate of interest. The investment rate of interest is irrelevant, as bond finance does not in this case affect private investment.[38]

In concluding this section, we return to the issue of the equivalence of debt and tax finance. We have seen that, if investment in the capital market is a permissible use of tax revenues and if society in its current spending behavior is insensitive to the size of the public debt, then both debt and tax finance of public expenditure *for any purpose* must bear an inordinate cost. A direct implication of that cost is that taxes should be increased with the proceeds being invested in the capital market until the marginal social collection cost just balances the benefit of expanding the distorted activity (privately financed investment). Neglecting the burden of tax-collection costs, the benefit of this operation, measured in terms of present value (at constant real output), is $[(\rho/r)(\partial I/\partial r) + (\partial C/\partial r)]\, dr = [\theta\rho/r + (1 - \theta)] f_r\, dr$ per dollar of debt retirement. The resultant change in the interest rate is given by subtracting equation (9) from (10) which reduces to $f_{y^e}(1 - \phi')/f_r$, and hence the benefit is $\alpha f_{y^e}(1 - \phi')$ per dollar of debt retirement.[39]

But this result carries an immediate implication which, in a very real sense, brings us back to the true meaning of second-best analysis: a

[37] If, e.g., $r = 0.05$ with ρ and θ being such that $\omega = 0.10$, then $\alpha = 2$, implying that bond-financed consumption expenditures must register an excess of benefits over costs of at least 100 percent.

[38] This is equivalent to the case analyzed earlier in this section in which all bonds are voluntarily purchased at the expense of consumption.

[39] Equations (9) and (10) were derived assuming $\partial y^e/\partial b = \partial y^e/\partial t = -1$. The interest-rate effect obtained above for tax-financed debt retirement requires only that $\partial y^e/\partial t - \partial y^e/\partial b = 0$; hence it is consistent with constant real output.

government that can vary tax revenues (and hence tax rates) at will obviously also has the power to eliminate distortions in the capital market. It follows, then, that the case for a shadow price of public-sector expenditure (social rate of discount) in excess of unity (the consumption rate of interest) is fundamentally a case for tax reform. This point has received far too little emphasis in second-best treatments of the social opportunity cost of public funds.[40]

IV. A Cost-Benefit Analysis of Fiscal Policy

We now turn our attention to the social value of fiscal policy and its influence on the level of economic activity. The central point in what follows is that the appropriate measure of the efficiency of fiscal policy is not the magnitude of any fiscal multiplier but rather the social value to be attached to incremental output—current and future—attributable to fiscal measures. To maintain the framework of the preceding analysis, we will focus primarily on the impact of bond finance but also investigate the differential effects of other modes of fiscal policy.

To confine the analysis to pure fiscal policy, the passive money introduced in Section III is replaced by a monetary policy independent of both public finance and public-sector expenditure. In particular, the nominal stock of money is taken as fixed. We assume further that real output can expand or contract freely at a constant price level in response to fiscal stimulus, subject only to the constraint that the demand for money equal the supply. We accept the dubious realism of this assumption in order to examine the case most favorable to fiscal policy.

Utilizing equations (4)–(8) and fixing $dt = 0$ and $db = dg$, we obtain the following expression for the change in expenditure and output: $dy^e = dy = f_{y^e}(dy + h'dg - \phi'dg) + f_r dr + f_g dg - h'dg + dg$. As the nominal (and hence real) money stock is constant, equality of demand for and supply of money implies: $dr = -(L_y/L_r)dy$, where L_y and L_r are the partial derivatives of the demand for (real) cash balances with respect to output and the rate of interest, respectively. Combining these two equations, we have:

$$dy/dg = \mu = [(1 + f_g') + f_{y^e}(h' - \phi')]/(1 - f_{y^e} + f_r L_y/L_r), \tag{16}$$

where $f_g' = f_g - h'$. The response of the interest rate to changes in public-sector spending is:

$$dr/dg = \mu_r = -(L_y/L_r)(dy/dg) = -(L_y/L_r)\mu, \tag{17}$$

and $f_r\mu_r = -(f_r L_y/L_r)\mu = -\beta\mu$ where β is a positive constant. We can now evaluate fiscal policy in terms of this simple model. The approach

[40] Harberger (1973*b*, p. 121, n. 12) makes essentially this same point, although in a rather different context.

uses the decline in privately financed expenditure, evaluated at shadow prices, to measure the "cost" of the project output. This cost can, of course, be negative as both private and public output could expand. We emphasize at the outset that this measure of cost does not include the nonmarket opportunity cost of the resources engaged to produce an increase in global output.

The change in output (Δy) associated with an increase in public-sector spending (Δg) is obviously divided between global consumption and global investment. The change in privately financed expenditure is Δf less the change in the value (to users) of the public-sector output, $\Delta h = h'(\Delta g)$. In terms of (the differentiated form of) equation (7) of Section III:

$$dy = (df - h'dg) + dg. \qquad (7')$$

The term in parentheses is the change in privately financed consumption and investment, which expands to:

$$df - h'dg = f_{y^*}dy^* + f_r dr + f_g dg - h'dg$$
$$= [f_{y^*}(dy + h'dg - \phi'dg) - h'dg] + f_r dr + f_g dg. \qquad (18)$$

The bracketed term of equation (18) is the change in privately financed spending directly induced by the change in perceived income, corrected for the direct substitution of public-sector for privately financed output; this change in output is assigned the shadow price of α^* as defined in Section III. The second term of equation (16) is privately financed spending brought about by the interest-rate effect and is assigned the usual shadow price of α. The final term captures spending induced by a broadening of the opportunity set, whose shadow price is α', also defined in Section III.

The shadow price of the project output, h', is now defined as α_3: $\alpha_3 = \alpha^*[f_{y^*}(dr/dg) + f_{y^*}(h' - \phi') - h'] + \alpha f_r(dr/dg) + \alpha' f_g$. Again introducing a negative sign into each term in order to signify cost, substituting equation (17) into the above, and rearranging, we obtain the shadow cost of the project output:

$$\alpha_3 = (\alpha\beta - \alpha^* f_{y^*})\mu + \alpha^*[h' + f_{y^*}(\phi' - h')] - \alpha' f_g. \qquad (19)$$

A definitive evaluation of α_3 is probably impossible, owing to the large number of coefficients to be evaluated; but some illustrative calculations, based upon plausible orders of magnitude, yield rather startling results.[41] An assumed value for β can be obtained by writing that coefficient as follows: $\beta = f_r L_y / L_r = (f/y)\eta'_{fr}\eta_{my}/\eta'_{mr}$, in which η and η' indicate

[41] To evaluate eq. (19) we need estimates of ρ, r, θ, θ', θ^*, f_g, f_{y^*}, h', ϕ', and β. Not only is the estimation problem formidable, but also the magnitude of some of the coefficients will respond to the specific nature of the public-sector expenditure.

TABLE 1

SOCIAL COSTS OF DEBT-FINANCED EXPENDITURE

ϕ'	f_g	μ	α_2	α_3	$\alpha_2 - \alpha_3$
1	0	0	α^*	α^*	0
1	1	1	$\alpha - \alpha' + \alpha^*$	$\alpha - \alpha'$	α^*
0	0	1	α	$\alpha - \alpha^*$	α^*
0	1	2	$2\alpha - \alpha'$	$2(\alpha - \alpha^*) - \alpha'$	$2\alpha^*$

elasticity and semielasticity, respectively, and m is the demand for (real) cash balances.[42] The value of (f/y) is approximately unity (exactly so if $h[g] = g$) and the income elasticity of demand for real cash balances is widely thought to be at least unity—some estimates are well in excess of unity. It is plausible that the semielasticity of expenditure and of the demand for money with respect to the interest rate are of the same order of magnitude; therefore we find it reasonable to expect that β has a value of at least unity.

We also assume that f_{y^*}, the marginal propensity to spend, is unity. This rather high value of f_{y^*} reflects a liberal allowance for accelerator effects and in addition, for immediate purposes, has the side benefit that h' disappears from the multiplier μ. Evaluating α_3 accordingly, we obtain:

$$\alpha_3 = (\alpha - \alpha^*)(1 + f_g - \phi') + \alpha^*\phi' - \alpha' f_g. \qquad (19')$$

We now proceed to obtain the range of values for α_3 in terms of α, α', and α^* for various limiting values of ϕ' and f_g. The coefficient ϕ' clearly lies in the zero–unity interval and we make the same assumption with respect to f_g, although clearly only an extraordinary project will create an equal amount of private-sector expenditure in response to a broadening of the opportunity set for consumption and/or investment. Taking all combinations of the limiting values for ϕ' and f_g, we have calculated μ, α_2, and α_3 in terms of α, α', and α^*; the results are presented in table 1.

The shadow price α_3 cannot exceed α_2 as the latter is defined for constant (contemporaneous) output, whereas the former permits global consumption and investment to respond to fiscal stimulus. The two differ, of course, by exactly the social value, α^*, of the increase in global spending. It is again emphasized that neither α_3 nor the difference between α_3 and α_2 represented in table 1 include the social value of the resources required to produce the additional output.

Further evaluation of α_2 and α_3 requires knowledge of α, α', and α^*, all of which range from unity to ρ/r. Given the distortions in the capital market in most countries, it is plausible that ρ is at least twice r and hence

[42] $\eta_{xz} = \partial \ln x / \partial \ln z$, and $\eta'_{xz} = \partial \ln x / \partial z$.

the range for these coefficients is assumed to be from one to two. Recalling the definitions of these coefficients, we can now write them as: $\alpha = 1 + \theta$; $\alpha' = 1 + \theta'$; and $\alpha^* = 1 + \theta^*$.

In the case of public-consumption projects it is reasonable, as was explained in the previous section, to assume that θ^* is approximately zero. Further, such projects will be likely to offer but limited scope for new, privately financed investment; hence θ' is also approximately zero. For such projects, then, we assume that both α' and α^* are approximately unity. If in addition θ is approximately unity (consumption very interest inelastic), it follows that α_3 is approximately unity for all four cases contemplated in table 1. As α_3 is to be interpreted as the *minimum acceptable social value of the direct output of a project*, we conclude that consumption-oriented public spending must at least "pay its own way," in the sense that the value of the output in the eyes of the beneficiaries must be at least as large as the costs (measured at market prices), even if the shadow price of resources is zero.[43] To the extent that resources required to produce additional output have a positive social cost, the value of the output of the project must rise accordingly.

Investment projects with salable output are less likely to imply future tax obligations for debt service and retirement, and hence ϕ' is likely to be small or even zero for such projects, indicating that privately financed spending $(f - h)$ will be affected only through the multiplier effect; in this case we expect that θ^* is positive but quite small. In addition, as such projects are more likely to improve private-sector investment opportunities, θ' is also positive. Referring to rows 3 and 4 of table 1, we conclude that α_3 is less than unity for investment projects.

Anyone can, of course, construct many different sets of values for α_2 and α_3; in particular, it is clearly possible to create cases more favorable to fiscal policy. For example, the more elastic is the demand for money and inelastic is expenditure with respect to the interest rate, the smaller will be the magnitude of β (but the larger the multiplier) and hence the greater is the possibility that the term $(\alpha\beta - \alpha^* f_{y_s})$ will make a negative rather than a positive contribution to the social cost. But these empirical questions are beyond the scope of the present analysis. What we have demonstrated is that the global output effect of fiscal policy can lower the shadow price of project inputs (and likewise the social opportunity cost of capital), but nevertheless that shadow price is unlikely to reach zero, as is implicitly assumed in most simple multiplier approaches to the evaluation of fiscal policy.

[43] The prices here referred to are already corrected for the traditional distortions such as taxes and monopoly/monopsony, but are not corrected for differences between nonmarket opportunity cost and market prices arising from involuntary unemployment.

V. The Social Cost of Labor: Little and Mirrlees

We now consider the novel concept of the shadow wage rate developed by Little and Mirrlees (1969, 1974); we include this because, as in the case of the social rate of discount, the Little-Mirrlees (LM) shadow wage concept depends crucially upon the distinction between the consumption and investment rates of interest. The LM shadow wage is the sum of the direct opportunity cost of labor employed in the project (taken to be the rural marginal product of labor) and the welfare loss arising from project-induced increments to consumption at the expense of (potential) investment. This welfare loss is based upon their argument that consumption is a distorted activity coupled with their naive (but traditional) Keynesian assumption that public-sector employment enlarges the aggregate wage bill and consumption in equal amounts. In short, public-sector employment not only reduces private-sector output but also expands an activity (consumption) in which social cost exceeds social value. In this way, the LM analysis appears to be quite similar to that of Harberger, Marglin, and Sandmo-Dreze; it differs, however, in that LM infer the consumption rate of interest from the rate of change of (per capita) consumption and in that they inadvertently—and erroneously—use the (present value of) investment rather than current consumption as the *numeraire*.

Turning first to the consumption rate of interest, Little and Mirrlees argue that if per capita consumption is constant, the consumption rate of interest is zero. Here they are either implicitly assuming that the pure rate of time preference is zero or they have fallen into the error of associating a zero rate of interest with the stationary state (1969, chap. 3). It is elementary that when the pure rate of time preference is stable and with instantaneous utility depending on instantaneous consumption, the rate of change of marginal utility equals the difference between the pure rate of time preference and the consumption rate of interest, permitting the latter to be positive even if consumption (and hence marginal utility) are constant.[44] Evidently, then, secular stagnation of per capita consumption is insufficient to distort consumption as an activity and hence generate a difference between the shadow wage and the forgone marginal product of labor.

[44] Define $c(t)$ as (per capita) consumption, $z(t)$ as the maximum level of consumption that can be maintained indefinitely in the basis of the existing capital stock, $\dot{z}$ as the time rate of change of z, and $u[c(t)]$ as the twice-differentiable utility function for which $u' \geq 0$ and $u'' < 0$. The Euler condition for a maximum of discounted utility:

$$\int_0^\infty u[c(t)]e^{-st}\,dt,$$

subject to $\dot{z}(t) = r[z(t) - c(t)]$, is $su' - \dot{u}' = ru'$, hence, $\dot{u}'/u' = s - r$. If s is an increasing, and r a decreasing, function of z, the result is more complex: $\dot{u}'/u' = s - r + (\dot{z}/z)(z\eta_{s,z} - \eta_{r,z})$. As the bracketed term is positive, an approach to the stationary state ($\dot{u}' = 0$) is possible.

But the central result obtained by LM does not depend upon the reason for a wedge between the investment and consumption rates of interest, only that a gap exist. Given the widespread taxation of income in general and income from capital in particular, their concept must be examined further.

The LM analysis rests on their contention that (at least part of) the project-induced increment to the aggregate wage bill is consumed when it could be invested.[45] Defining units of labor such that its opportunity cost (designated "m" in the LM equations) is unity, their formulation of the shadow wage is:

$$w_s = 1 + (c - 1)[(s - 1)/s], \tag{20}$$

where c is the per-worker consumption by project employees and s is the value of investment relative to consumption (i.e., the value of the consumption stream, discounted by r, that can be generated by investing \$1.00 today) (Little and Mirrlees 1969, p. 162). As it is assumed that $(c - 1)$ is positive, the shadow wage exceeds the opportunity cost of labor whenever ρ exceeds r; note, however, that the LM shadow wage is bounded by the marginal product of labor in agriculture (unity) and the project wage (the assumed upper limit for c).[46]

An obvious and natural interpretation of equation (20) is to treat $(c - 1)$ as the change in the level of a distorted activity and $(s - 1)/s$ as the distortion. The coefficient s is, of course, equal to ρ/r—the present value of a perpetual stream of consumption equal to ρ discounted by r—and thus is equal to Marglin's α when saving is completely interest inelastic ($\theta = 1$), which would appear to be the implicit LM assumption. Thus we can write equation (20) as:

$$\begin{aligned} w_s &= 1 + (c - 1)[(\rho - r)/\rho] \\ &= 1 + (c - 1)[(\alpha - 1)/\alpha]. \end{aligned} \tag{21}$$

It is clear that LM are measuring wages in terms of current consumption goods, but from equation (21) it is equally clear that they are normalizing the distortion $(\alpha - 1)$ by α, the consumption value of investment and are thereby erroneously using investment as the *numeraire* in evaluating the distortion.

The correct measure for the shadow wage emerges from our approach in Section III in which consumption is the *numeraire*. Given $\theta = 1$, the externality associated with consumption is $-(\rho/r - 1)$, which is simply the product of the associated change in investment (minus one) and the present value of the distortion associated with that activity measured in terms of consumption goods. The negative sign indicates that consumption reduces potential welfare and hence can be dropped when we view the externality as a cost. Thus the corrected LM shadow wage is:

$$\begin{aligned} w_s &= 1 + (c - 1)(\rho/r - 1) \\ &= 1 + (c - 1)(\alpha - 1) \\ &= 1 + (c - 1)(s - 1), \end{aligned} \tag{22}$$

which differs from equation (20) only in that the distortion is measured in current consumption rather than investment.

Now s, the value of investment measured in consumption, is presumably at least unity (cases where $\rho < r$ are uninteresting) but, since it has no upper limit, the correct representation of the LM result is unbounded from above. Thus the corrected result does not possess the highly convenient but curious property of the original formulation (eq. [20]) that, as the consumption rate of interest approaches zero, the distortion approaches unity and the shadow wage approaches the project wage (c). With the revised formulation (eq. [22]), both the distortion and the shadow wage increase without limit as r approaches zero. This is as it should be; if in fact there is no time preference and if there exist opportunities to invest at a positive rate of return, then to detract from such investments poses a cost the present value of which can be arbitrarily large.[47]

Finally, the corrected result (eq. [22]) does not depend upon the value of θ. Although LM argue that all increments in consumption are at the expense of investment, they posit no realistic argument to validate the assumption. If the project is unsubsidized, measured output of the economy is increased by the return to capital plus the excess of the wage bill over the marginal product forgone elsewhere, permitting an increase in consumption with no reduction in investment. But the wage bill must be financed by selling the output, increasing taxes, or selling bonds. As is clear from the analysis of Section IV, each of these alternatives has effects on both consumption and investment, and hence we should replace $(c - 1)$ in equation (22) with the net change in consumption, or alternatively attach to $(c - 1)$ an externality which reflects the fact that some fraction of the funds generated to pay the wage bill come at the

[45] The rise in the aggregate wage bill comes about because the project wage, for unspecified reasons, exceeds the marginal product of labor.

[46] It is unlikely to be unfair to infer that the LM adjustment to the forgone marginal product of labor in alternative pursuits was inspired by a clearly perceived need for a shadow wage rate of an order of magnitude similar to that of the market wage. As Harberger (1973a) has ingeniously pointed out, the difference between the market and the shadow wage is, from the social point of view, a transfer from capital to labor and thus is part of the (social) return to capital. If that difference is large, as occurs when the social cost of labor is the (assumed zero) marginal product of agricultural labor, the discount rate becomes very high; indeed so high that the more capital-intensive, showpiece projects so much in vogue in certain backward countries are impossible to justify with conventional cost-benefit analysis.

[47] Note, of course, that if the capital created by the project constituted a permanent addition to the capital stock, the value of the project itself is infinite.

expense of investment. Clearly θ is that fraction, so we obtain: $w_s = 1 + (c - 1)\theta(\rho/r - 1) = 1 + (c - 1)(\alpha - 1)$, which is identical to equation (22).

Note that when $c = 2$ (project-induced consumption is exactly unity), we obtain the Marglin shadow price of capital inputs: $w_s = \alpha$. This result should surprise no one; the LM approach penalizes a project for employment of labor which increases consumption at the expense of (potential) investment. The Marglin approach penalizes the employment of capital because, in the absence of a specific reinvestment strategy, the depreciation of that capital will be consumed at the expense of investment.

In summary, Little and Mirrlees have confused the debate over the social cost of labor by (*a*) failing to posit sufficient conditions for a shadow wage different from the market wage and (*b*) incorrectly defining the shadow wage even when those conditions are met.

VI. Conclusions

Since this paper has covered a great deal of territory, we shall devote this concluding section to a summary and recapitulation of our major findings.

1. Within the narrow confines of traditional cost-benefit analysis, given that such analysis is based on (*a*) the consumption rate of interest as the only truly relevant rate of discount for calculating the present value of future net-benefit flows directly attributable to public-sector projects, but also on (*b*) the concomitant recognition of the fact that, with the almost universal prevalence of capital market distortions, the investment rate of interest exceeds the consumption rate and therefore that the social opportunity cost per dollar of private investment forgone in order to finance the public project exceeds $1.00, the only possible result as to the social rate of discount (ω) is a weighted average of ρ and r. The requisite weights are, of course, the shares in which each dollar of financing for public-sector investment projects comes, respectively, at the expense of privately financed investment (θ) and private consumption ($1 - \theta$). Our demonstration of this result mirrors (and simplifies) the previous analyses of Harberger and of Sandmo and Dreze.

2. The net-present-value criterion, using ω as the social rate of discount, and Marglin's well-known "alternative" criterion of discounting solely by the consumption rate of interest, but adjusting the initial capital cost by the shadow price of capital-goods inputs (α), were shown to be exactly equivalent (*a*) when the public project generates a perpetuity, and (*b*) if we graft a capital market onto Marglin's model in order to impart a modicum of economic behavior to the shadow-price procedure, particularly with respect to the definition of θ. In at least these terms, then, a great deal of the continuing debate that has taken place between the partisans of the two approaches has been meaningless.

3. However, these approaches do significantly differ for public projects with less than infinite lives; yet the crucial difference lies not in the rate at which future net-benefit flows should be discounted—the focus of the last 12–15 years of debate in the literature—but rather in their divergent, implicit assumptions as to the general public's treatment of the depreciation deriving from governmentally financed investment. The philosophy of Marglin's basic model denies a distinction between depreciation and net output; hence, contrary to the usual interpretation, his criterion is far more stringent than is the social-discount-rate approach for finite-lived projects. On the other hand, the criterion based on ω as the appropriate rate of discount implicitly assumes that, one way or another, the general public recognizes public-sector depreciation as such and therefore attempts to save rather than consume it; through subsequent capital-market effects, the attempt will be partially unsuccessful, such that the end result will be a global stock of capital that is unaltered (for marginal projects) after the complete execution of the project. Acceptance of the Marglin position requires the heretofore lacking explanation as to why private expenditure behavior should be so widely divergent with respect to private-sector versus public-sector depreciation, in view of the fact that the option of consuming a portion of society's capital stock always exists with or without government expenditures. On the other hand, acceptance of the social discount rate approach requires a conviction that it is in fact feasible for the public to distinguish between gross and net output of public projects.

4. The foregoing thus represents one fundamental reinvestment issue that has not received previous attention in the literature. The second, and more widely recognized, question concerns the effects that arise from the possibility of reinvesting all or part of the *net* throw-off from public-sector projects. Owing to the fact that an acceptable public-sector investment must cause global output to be greater than it would have been in at least one subsequent period, we find that the option of investing all or part of that incremental output creates a qualitatively (though in all likelihood not a quantitatively) important adjustment to the social opportunity cost of capital as envisaged by Harberger and by Sandmo and Dreze.

5. When we extend the analysis to a broader, macroeconomic framework, it becomes clear (eq. [12]) that the uniqueness of ω—as is asserted by Harberger and Sandmo-Dreze—depends upon three quite stringent conditions: (*i*) real output is exogenously determined during the investment period; (*ii*) there is no direct effect of project execution on privately financed expenditures; (*iii*) the general public exhibits equal perceptiveness to the value of public output, on the one hand, and to the present value of future tax liabilities arising from governmental bond finance, on the other. Since, even given *i*, there is no obvious reason to accept *ii* and *iii* as generally valid, we relax them in Section III in order to explore

the implications for the general shadow price of public finance. First, we find that, although both the social discount rate and the shadow price of inputs continue to generate equivalent investment criteria, the latter is the more easily extended approach, being applicable to both investment and consumption projects. More importantly, as our equations (14) and (15) indicate, relaxing conditions *ii* and *iii* has potentially dramatic effects on either social-evaluation concept. There is no presumptive case to be made for the uniqueness (across varieties of public expenditures) of either concept.

However, given that this nonuniqueness depends, in turn, on rather poorly estimable differences—$(\alpha - \alpha^*)$ and $(\alpha - \alpha')$ on the one hand, and $(\theta - \theta^*)$ and $(\theta - \theta')$ on the other—it can be argued that assuming conditions (*i*)–(*iii*) is not inappropriate for defining α or ω, especially since the complications so conveniently eliminated thereby can be reinstated through the incorporation of other (perhaps "external") costs and/or benefits for each project. Nevertheless, our analysis highlights the importance of identifying and explicitly including all such additional effects within the project-evaluation procedure.

6. One of the important conclusions arising from the macroeconomic framework of analysis is that in general there is no basis for asymmetrical treatment of publicly financed investment and consumption projects. In other words, if we are justified in discounting returns to investment projects at a rate higher than the consumption rate of interest, then we are also justified in charging a shadow price for consumption-project inputs that is higher than the market price. (Ironically, this conclusion follows directly from the Marglin approach as well.) This conclusion is of considerable quantitative significance as the currently encountered arguments for discount rates in the neighborhood of 10 percent (in real terms) also justify a surcharge on consumption project inputs in the neighborhood of 100 percent!

7. In Section IV we relax the assumption of exogenously determined real output in order to evaluate, from the social cost-benefit viewpoint, the effects of anticyclical fiscal policy. The requisite expression for this evaluation (eq. [19]) is too complex for precise estimation, but by choosing plausible values (or ranges) for the underlying parameters we conclude that the case for anticyclical fiscal policy is anything but strong. Even under the most favorable (Keynesian) assumption—an infinitely elastic supply of resources at a zero social cost—we find that it will generally be true that both investment and consumption projects must "pay their own way" in the sense that project-specific benefits must be sufficient to justify project-specific costs.

8. Finally, almost as a digression in Section V we briefly consider—and quickly dispense with—the Little-Mirrlees notions concerning the shadow price of labor. While the Little-Mirrlees analysis can be recast to

conform with the conventional treatment of distortions, their case for consumption being a distorted activity is lacking. In addition, they erroneously use investment as the *numeraire*, thus compromising both the qualitative and quantitative aspects of their central result.

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